

Estimated Ground Motion From the 1994 Northridge, California, Earthquake at the Site of the  
Interstate 10 and La Cienega Boulevard Bridge Collapse, West Los Angeles, California

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Abstract

We have estimated ground motions at the site of a bridge collapse during the 1994 Northridge, California, earthquake. The estimated motions are based on correcting motions recorded during the mainshock 2.3 km from the collapse site for the relative site response of the two sites. Shear-wave slownesses and damping based on analysis of borehole measurements at the two sites were used in the site response analysis. We estimate that the motions at the collapse site were probably larger, by factors ranging from 1.2 to 1.6, than at the site at which the ground motion was recorded, for periods less than about 1 sec.

Introduction

During the **M** 6.7 Northridge, California, earthquake of 17 January, 1994, bridges at two sites along the interstate highway I10 corridor in the western part of Los Angeles collapsed or suffered major damage (Caltrans, undated). Both sites at which the bridges suffered major damage or collapse are underlain by considerably thicker Holocene deposits than those underlying nearby bridges that suffered minor to moderate damage. We focus on the intersection of I10 with La Cienega Boulevard, at which overpass bridges collapsed (we refer to this as the “I10 site” or sometimes as “I10”); other similarly built overpasses along the highway within several km did not collapse. There are geological reasons to believe that the near-surface materials are softer at the collapse site than at nearby sites (“ciénaga” means “marsh” in Spanish). The I10–La Cienega Boulevard intersection is located approximately 24 km southeast from the epicenter of the Northridge earthquake

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(Figure 1). No strong-motion records were obtained at this site during the Northridge mainshock. The nearest strong-motion instrument that recorded the mainshock, Saturn Street School (USC91), is located 2.3 km northeast of the bridge site (Anderson *et al.*, 1981; Figure 1). Subsequent to the earthquake several boreholes were drilled at the I10 site (Darragh *et al.*, 1997) and one borehole was drilled at Saturn Street School (referred to as the “Saturn site” or sometimes simply as “SAT”). The boreholes have been logged using various methods. We interpreted the borehole measurements to obtain near-surface shear-wave slownesses and damping, and we used this information to estimate the ground motion during the mainshock at the I10 site by correcting the recorded ground motion at Saturn Street School for the relative site responses at the two sites, using both linear and equivalent-linear approximations to nonlinear soil response calculations. We find that the ground motions at the I10 site were probably larger, by factors ranging from 1.2 to 1.6, than at the Saturn Street School site for periods less than about 1 sec, although inherent spatial variability does not allow us to be certain of this. We speculate that this difference in ground motion contributed to the localized collapse of the bridges at the I10 site.

### Near-Surface Slownesses and Attenuation

The basis for the estimates of ground motion at the I10 site is to deconvolve the recorded motion at Saturn Street School by the local site response, and then convolve this input motion with the site response at the I10 site. This procedure requires shear-wave slownesses and attenuation beneath both sites. In this section we describe how the models used in the calculations were constructed.

We use slowness rather than velocity (the two are reciprocals of one another), because differences in site amplification are most sensitive to differences in the near-surface seismic velocities; plots of slowness emphasize these near-surface differences better than do plots of seismic velocity. Plots of seismic velocities tend to be dominated by the higher velocities in a profile, and what may appear to be significant differences in velocities between two sites are often not reflected in differences in site response. In general, the amplification will be higher at sites with larger slowness near the surface. Furthermore, most slownesses are estimated directly from data as the slope of a line fit to travel time as a function of depth; seismic velocity values traditionally reported are simply the inverse of the slopes of the fitted lines.

*Shear-Wave Slownesses.* We used seismic slownesses derived from measurements made using two borehole logging methods: surface-to-borehole (“s2b”) logging and suspension logging. We did the s2b logging and are reporting the results for the first time here. The s2b method (Warrick, 1974) uses recordings on a transducer clamped at various depths

in the borehole using a surface source described by Liu *et al.* (1996). A record section is constructed of the recorded waveforms, and first arrival times are picked from the resulting record section; these arrival times are fit using a model with constant-slowness layers. Details of the measurement and interpretation methods are given in the U.S. Geological Survey Open-File Reports describing the results from many boreholes (e.g., Gibbs *et al.*, 2000). For the I10 hole the waveforms are very clean (Figure 2), making it easy to pick the first arrival times. The lithology, SP logs, resistivity logs, and derived shear-wave velocity for the I10 site are given in Figure 3; tables of the shear-wave and the compressional-wave velocities are given in Boore (2003).

Results from the suspension logging method were obtained from measurements and analyses performed by the ROSRINE project (<http://geoinfo.usc.edu/rosrine>) and by the California Department of Transportation (Caltrans) (C. Roblee, written commun., 1999). The suspension logging method uses a probe, containing both a source and receivers, that is lowered into the borehole (Nigbor and Imai, 1994). Interval slownesses are obtained between the receivers, 1 m apart, for a series of depths down the hole. The slownesses from the ROSRINE project at the I10 site are for depths between 26 m and 278 m (Figure 3); the Caltrans logging provided slownesses from 1.5 m to 95.6 m (Figure 4a). At the Saturn site the ROSRINE measurements were made between 1.5 m and 97 m; these results are given in Figure 4b. Caltrans measurements were not made at the Saturn site.

The suspension logging data are basically point estimates of the slowness, and they do not extend to the surface. For purposes of computing site amplifications, however, it is desirable to have a model of seismic slownesses made up of a stack of constant slowness layers. We have derived such models from the suspension logging data by computing the effective slowness for a set of depths corresponding to a layered model. The travel time across each layer was computed using

$$t_n = \sum_{i=1}^n s_i d_i, \quad (1)$$

where  $s_i$  are the slownesses from the suspension logging data,  $d_i$  is the spacing between suspension logging measurements, and  $n$  is the number of measurements in the layer. The equivalent slowness for the layer was computed from

$$s_{lyr} = \frac{t_n}{d_{lyr}}, \quad (2)$$

where  $s_{lyr}$  is the average slowness in the layer, and  $d_{lyr}$  is thickness of the layer. The missing top 1.5 m from the suspension logs was assumed to have the value of slowness measured at 1.5 m.

We constructed four models from the available suspension data: two models at I10-La Cienega and two at Saturn Street School. The models differ only in the choice of layering. Layering was chosen based on subjective inspection of the suspension logging data for each site separately; this layering generally has more detail than from interpretations of the s2b logging and the resulting models are termed the “cmplx” models. In addition, the layering of the model derived from the s2b measurements at I10 was used for averaging the suspension logging data at both the I10 and Saturn Street School sites; these models are termed “s2b” models. A comparison of the slownesses for these layered models and the suspension logging data are given in Figure 4.

For site response calculations it is desirable to have a velocity model that extends to depths great enough to influence the site response at the lowest frequencies of interest. We want to compute site response for frequencies down to about 0.5 Hz; for the I10 site, the maximum depth (250 m) corresponds to a quarter wavelength at a frequency of 0.5 Hz. Unfortunately, the velocities at the Saturn site are only available to about 100 m. Given the proximity of the two sites and the similarity of their slownesses below about 12.5 m (Figure 4), however, we decided to assume that the slownesses at depths greater than 97 m at the Saturn site are the same as at the I10 site. This assumption affects frequencies less than about 0.9 Hz.

A comparison of all layered models is given in Figure 5, from which it can be seen that the slownesses near the surface at I10 are consistently higher than at Saturn Street School. (The models, in terms of velocity, are tabulated in Table 1). In addition, the suspension logging slownesses at I10 are higher than from the s2b models. Finally, the models for both I10 and Saturn Street School are similar at depths below about 12.5 m. The site amplifications shown later will reflect all of these features.

The larger  $S$ -slowness at the I10 site reflects the appreciable thickness (10 m) of rather soft Holocene alluvium at that site. These sediments were deposited when the sea level rose as the last ice age abated, mainly in the interval from about 13,000 to 6,000 years before present. In contrast, Saturn Street School is located on the northern margin of the Ballona Creek flood plain where the Holocene deposits are very thin, persistent swamps were absent, and the Holocene overlies moderately dense Pleistocene alluvium—a distinctively different near-surface site condition than at the I10 site.

*Damping Factor  $D_s$ .* The site amplification functions used in our deconvolution/convolution procedure require, in addition to shear-wave slowness, an estimate of the damping factor  $D_s$ . In seismology, damping is more often measured by the quality factor  $Q$ , although the damping factor is the more natural quantity to use. The two factors are related by the

simple equation  $Q_s = 0.5/D_s$ . The quality factor  $Q$  is defined as  $2\pi E/\Delta E$ , where  $\Delta E$  is the energy lost through anelastic processes in a cycle of deformation and  $E$  is the peak energy stored in the cycle (Aki and Richards, 1980). Loosely speaking,  $Q$  is the number of wavelengths of propagation required for anelastic attenuation to reduce the amplitude of a wave by a factor of  $e^{-\pi}$ . We measured the shear-wave damping factor  $D_s$  by the method described by Gibbs *et al.* (1994) with minor modifications. Briefly, a preliminary value of  $D_s$  for each frequency  $f$  is obtained by correcting the natural logarithm of the shear-wave spectral amplitude for geometric spreading and for the effects of changes in the shear-wave impedance and then regressing the corrected amplitude value against shear-wave travel time. The preliminary value of  $D_s$  is obtained from the regression coefficient, which is equal to  $-2\pi f D_s$ . In order to correct for wave propagation effects, particularly reflections at layer boundaries, synthetic seismograms are generated using a computer program written by Herrmann (1996). This is a complete wave-propagation program that uses wavenumber integration and a layered earth model; the calculations include near-field terms and all interbed reflections. Synthetic seismograms are computed using the same slowness model as used in the analysis of the data and a damping value that is an average of those from the initial analysis. The synthetic seismograms are processed in the same way as the recorded seismograms and the difference between the derived damping and the value used in the calculations is used to correct the values from the analysis of the data. In all cases the corrections derived from the synthetic seismograms were small. Our computer programs for calculating  $D_s$  have an option that permits us to impose the condition that  $D_s$  is independent of frequency and calculate the value that best fits the data at all frequencies considered.

Applying this method to the s2b data from the I10–La Cienega site (shown in Figure 2) gives the results shown in Figure 6. There is relatively little frequency dependence over the range of the measurement, and the average  $D_s$ , from the frequency independent assumption over the depth range from 0 to 220 m, is 0.012. This value of  $D_s$  is slightly low compared to those we have obtained at other sites in California with comparable velocities and fine-grained soils; these other dampings are generally between 0.014 and 0.020. We assume that the damping factor of 0.012 also applies to the Saturn site. The damping factor depends on soil type as well as overall average velocity (unpublished results by the authors of this paper); according to the geotechnical logs available from <http://geoinfo.usc.edu/rosrine>, as well as shown in Figure 3, the soils under both sites are similar, being a mix of clays, silts, and sands. For this reason we feel justified in using the same damping for both sites.

#### Estimate of Ground Motions at the I10–La Cienega Site

The procedure for estimating the ground motion at the I10–La Cienega site from the

recorded motion at Saturn Street School is based on deconvolving the observed motion at the Saturn site to obtain the equivalent input motion at the base of the 250 m stack of layers, and then using this motion as input into the 250 m stack of layers beneath the I10 site to compute the surface ground motion at that site. We have done the calculations assuming both linear and an equivalent-linear approximation to nonlinear response. The procedures and results of each are described in turn.

*Linear Calculations.* By assuming that the system is linear, the step of deriving the deconvolved time series beneath the Saturn site can be dispensed with, and the Fourier spectra of the surface motion at the I10 site can be written as

$$A_{I10}(f) = \frac{S_{I10}(f)}{S_{Sat}(f)} A_{Sat}(f), \quad (3)$$

where  $A_{I10}(f)$  and  $A_{Sat}(f)$  are the Fourier spectra of the estimated motion at the I10–La Cienega site and the recorded motion at the Saturn Street School site, respectively (we use the 110 degree horizontal component motion at Saturn Street School in the analysis).  $S_{Sat}(f)$  and  $S_{I10}(f)$  are the site amplification functions (including amplitude and phase) at Saturn Street School and I10–La Cienega, respectively, relative to the motions for a model in which the top 250 m of the soils at each site have been removed (i.e., the outcrop motion for the halfspace beneath the stacks of layers). There is probably not a large impedance change in the vicinity of 250 m, so the results are not sensitive to the choice of depth corresponding to the halfspace. Impedance changes at greater depths (such as at the interface between Quaternary and Tertiary deposits) will affect motions at longer periods than of interest here. Site amplifications for several of the layered models are shown in Figure 7. The site amplifications were computed assuming *SH*-waves at an incidence angle of 10 degrees from vertical (the results are not sensitive to this choice). The inverse Fourier transform yields an estimate of the ground acceleration time series at the I10–La Cienega site. The motions at the I10 site are larger than at the Saturn site for frequencies between about 1 and 10 Hz, as expected from the comparison of the slownesses in Figure 5. If the waves were not damped, the motions at I10 would be larger than at SAT for frequencies higher than 10 Hz as well; this is not the case, however, because of damping. Even though the damping value (a constant value for all layers) was the same for both I10 and SAT, the travel time is greater in the near-surface materials underlying I10, and thus the effect of the damping on the high-frequency motions is greater at I10 than at SAT.

An important assumption in the analysis above (and the nonlinear analysis to follow) is that the input motion obtained from deconvolving the site response at Saturn Street School is the same as the input motion below the I10–La Cienega site. At first glance this seems to be a reasonable assumption, given the similarity of the soil profiles below 12.5 m (Figure 5),

the close separation of the two sites (2.3 km) compared to the distance to the source (about 24.5 km; Figure 1), and the similarity of the azimuths from the epicenter to the sites ( $318^\circ$  and  $323^\circ$  for the Saturn Street School and I10–La Cienega sites, respectively). Figure 8 shows the site amplification at both sites with the top 12.5 m of sediments removed. The two sites show very similar amplification with these sediments removed. We conclude the path of the earthquake wave-front (amplitude and frequency content) was essentially the same as it swept through the two sites. In fact, the assumption that the seismic wave input to the S-wave models was the same at 250 m depth beneath the two sites may be the weakest assumption in this article; we discuss this in a separate section, just before the conclusions.

*Nonlinear Soil Response.* Trifunac and Todorovska (1996) found evidence for nonlinear soil response out to a maximum distance of 20 km from the epicenter of the Northridge earthquake at sites with soft soil conditions (which they defined as having average slowness over the upper 30 m between 0.003 to 0.006 sec/m; the average slowness at I10 and SAT over the upper 30 m is about 0.0040 and 0.0033, respectively). They reported that at several strong-motion sites the peak horizontal amplifications were less than expected and attributed this to nonlinear soil response. Based on the relatively large peak acceleration and modest strain (as estimated from the peak velocity) recorded at Saturn Street School (station USC91, the station closest to the borehole site at I10–La Cienega), Trifunac and Todorovska (1996) do not include the Saturn site as one for which the soil response was nonlinear during the Northridge earthquake (Trifunac and Todorovska, 1996). On the other hand, the nearby I10 site has softer soils, with standard penetration blow counts from engineering borings of  $N \approx 10$  (geotechnical logs from <http://geoinfo.usc.edu/rosrine>). In addition, utility-pipe-breakage occurred close to the intersection of I10–La Cienega, suggesting possible nonlinear ground displacement (although no sand boils, the usual evidence cited for liquefaction, were observed [Stewart *et al.*, 1996]). Finally, a number of studies have found evidence for widespread nonlinear soil response during the 1994 Northridge earthquake (e.g., Field *et al.*, 1997, 1998). For these reasons we thought it prudent to derive surface motions at the I10 site assuming nonlinear soil response (at both sites).

We followed the procedure described in Cultrera *et al.* (1999), using an equivalent-linear approximation of nonlinear wave propagation to deconvolve the recording at Saturn Street School to derive the equivalent outcrop motions beneath the upper 250 m of the sediments. We used the program SHAKE91 (Schnabel *et al.*, 1972; Idriss and Sun, 1992) to deconvolve the motions at SAT and to propagate these motions through the 250 m of sediments beneath the I10 site to obtain the surface ground motions at the I10 site. Although not doing true nonlinear calculations, the SHAKE91 program is widely used to

compute the effects of nonlinear propagation, and the results are usually considered to give an adequate representation of truly nonlinear wave propagation, at least for frequencies up to about 10 Hz (W. Silva, pers. commun.). For economy of expression, from here on we refer to SHAKE91 output as being the result of doing nonlinear calculations. The modulus reduction and damping curves used in the analysis were those recommended by Silva *et al.* (1996) for use in the Los Angeles region for cohesionless soils. For a given range of depths, these curves have less nonlinearity than the generic curves contained in EPRI (1993). To be specific, for depths between 0 and 15 m, the EPRI curves for depths of 16 m to 46 m were used, and for depths greater than 15 m, the EPRI curves for the depth range 153 m – 305 m were used. We also did the analysis by scaling the motions at the Saturn site to approximate linear response; the results were similar to those we obtained by doing the frequency domain analysis described above.

*Results: Ratios of Response Spectra and Estimates of Motion at I10.* The result of both the linear and nonlinear calculations is a time series of ground acceleration at the I10 site, so time series are available at both the I10 and the Saturn sites. We thought that the most meaningful comparison of the motions would be provided by computing the response spectra for the motions at each site and then graphing the ratio of the response spectra from the two sites. We have done this and summarize the results in Figure 9. The shaded and hatched regions represent the range of ratios obtained using the six combinations of slowness models (three models are available at the I10 site, one being the model from the s2b logging, and two from fitting the suspension logging data to two different sets of layers, and two models are available at the Saturn site, corresponding to the different assumptions about layering used in averaging the suspension logging results). In all cases, the response at the I10–La Cienega site is greater than that at the Saturn Street School site for periods less than 1 sec. As expected from the slownesses shown in Figure 5, the largest difference is for the models based on the suspension logs. Even the smallest relative amplitude difference exceeds a factor of 1.2 for a wide period range, however. Interestingly, the nonlinear response predicts higher relative motions at the I10 site than the linear model for periods greater than about 0.3 sec. This is undoubtedly due to the softening of the near-surface sediments produced by the relatively high strains in the layers (the modulus reduction in the upper 12 m at I10 ranged from 0.36 to 0.51, with peak strains between 0.26 to 0.12 percent). The concurrent increase in damping (more than a factor of six) is not enough to offset the increase in amplification due to the increase in slowness of the sediments. The absolute ground motions estimated for the I10–La Cienega site are shown in Figure 10 for the range of linear and nonlinear calculations. We are not advocating that these motions be used in design, because of the effect of spatial variability, discussed next.



## Effect of Spatial Variability of Ground Motions

A number of studies find that waveforms of motions having frequencies above about 1 Hz rapidly lose coherence as station spacing increases, even for stations on sites with apparently similar surficial geology (e.g., Abrahamson *et al.*, 1991; Hough and Field, 1996). Other studies find a significant increase in the variability of ground-motion amplitudes as a function of station spacing (e.g., Abrahamson and Sykora, 1993; Steidl, 1993; Field and Hough, 1997). These latter studies are of particular importance regarding our assumption that the deconvolved motion beneath SAT is similar to the input motion under I10. Coherency is of less concern than overall differences in amplitude, because the various combinations of velocity models and the use of equivalent linear rather than true nonlinear calculations means that the detailed phasing of the input motion will not influence the overall level of ground motion calculated at I10. If the spatial variability were large enough it is possible that the input motion beneath I10 could have been so small relative to that beneath SAT that the surface motions at I10 could have been smaller than at SAT, even after amplification due to wave propagation through the sediments under I10. This is what we discuss in this section.

One way of assessing variability is to compare motions recorded at other stations. Three stations within a radius of 5 km of I10 recorded the 1994 Northridge mainshock (see Figure 1 for locations). Figure 11 shows the geometrical mean of the response spectra from these three stations. The spectra for Baldwin Hills (BWH) and Century City LACC North (CCN) have been corrected to the source-to-site distance of SAT (an average factor of 1.11 and 0.82 for BWH and CCN, respectively) using the equations of Boore *et al.* (1997). As shown in Figure 11, there are large differences between SAT and the other two sites. Velocities from P-S suspension logging are available from BWH, and these are very similar to those from Saturn Street School (SAT) (Figure 12). We could use these other motions as input to the velocity model under I10, and in terms of absolute motions the results would be similar to multiplying the ordinates in Figure 11 by the amplification factors in Figure 9 (of course, the nonlinear amplifications would not be the same, but in view of the variability and the fact that we are not advocating any particular motion for design, the overall level of motions would not change). Because SAT is the closest station, and also because the geographic setting is similar to that at the I10 site (not near the edge of mountains, as is CCN, and not in a hilly area, as is BWH), we consider the motions at SAT to be more appropriate than the other two motions as input. We recognize, however, that variability can still exist in the motions over a distance of 2.3 km. The rest of this section quantifies this variation and uses it to estimate a range of motions at I10.

Most studies of variability used smaller earthquakes than the 1994 Northridge mainshock and usually consider more than one earthquake; both of these factors can contribute to an overstatement of the variability expected for motions from the Northridge mainshock (e.g., Abrahamson and Silva [1997], Campbell [1997], and Sadigh *et al.* [1997] find that the scatter about the regression fits to the data decreases with magnitude, and Field *et al.* [1992], Liu *et al.* [1992], Field and Hough [1997], and Baher *et al.* [2003] find that the variability depends on source location). For these reasons we present results of a study of variability of peak accelerations from the Northridge mainshock alone. These results were published by Boore (1997), but because the report in which the study appeared is not widely available, we repeat the details of that study in the Appendix to this paper.

The measure of variability is the standard deviation of the difference of the logarithm of peak motions for all pairs of stations whose interstation spacing falls into a distance bin chosen such that 15 station pairs are included in each bin. The results are shown in Figure 11. Also included in that figure are results from small several small arrays, as well as the standard deviation about a regression curve for strong-motion data in the magnitude range 6.0 to 6.9. Because the application in this paper is to estimate the variability of one motion given another, the standard deviations have been increased by a factor of  $\sqrt{2}$  (this accounts for the fact that the observed motion might be lower or higher than the local mean of all data in a given interstation distance bin, and therefore the uncertainty of the predicted amplitude at another site, given the observed motion, should be greater than the variability about the local mean motion). Taken as a whole, the results clearly show that the variability increases rapidly with increasing station spacing. The results of Field and Hough (1997) are probably higher than those from the Northridge mainshock because they are studying a number of small earthquakes from a number of source locations. The most relevant results for our paper are from the Northridge mainshock and the SMART1 array, the latter because the site geology is relatively uniform, whereas site geology was not accounted for in the study of the Northridge mainshock peak accelerations. For the 2.3 km distance between I10 and SAT, the results in Figure 11 suggest that the variability of one motion given the other is  $10^{\pm 0.14}$  to  $10^{\pm 0.18}$ . (These variabilities are from observations at the ground surface, whereas we are interested in the variability of motions at a depth of 250 m. It is likely that a significant portion of ground-motion variability is due to changes in geology above this depth, but because we have no observations of spatial variability beneath the surface, we have used the conservative assumption that the spatial variability at the surface applies at depth.) For the important period range of 0.2 to 0.5 sec shown in Figure 9, the amplifications range from 1.1 to 1.6. Applying the variability factors to these amplifications results in the lowest amplification of 1.1 having a 68% chance of being between 0.7 and 1.7, and the highest amplification of 1.6 having a 68% chance of being between 1.1 and 2.4. The absolute ground motions in Figure 10 would be scaled

accordingly. The results of doing this are shown in Figure 13, which can be thought of as roughly showing the 68% confidence limits of the motion at I10. Thus the actual ground motions that occurred at I10 during the 1994 Northridge mainshock could have been somewhat lower than the motions at SAT, but overall it is more likely that they were amplified with respect to the motions at SAT.

## Summary and Discussion

We have made estimates of site amplification, pseudo relative velocity response, and acceleration values at I10–La Cienega during the Northridge earthquake. Critical to these estimates is establishing the similarities/differences between the Saturn Street School site, where records of the mainshock exist, and the site at the I10–La Cienega Boulevard intersection, where there are no mainshock records but where several bridges collapsed during the earthquake. Models based on shear-wave slowness measurements and lithology are compared at the two sites. Below 12.5 m depth the sites are nearly identical from a seismic response perspective. In view of the spatial variability in ground motion, we are not suggesting any particular motions to be used in engineering analysis of the bridge. We conclude, however, that the ground motions were probably higher at the I10–La Cienega site than at the Saturn Street School site, particularly in the range of periods from 0.1 to 1.0 sec, and that this difference is mainly due to the softer sediments (higher  $S$ -slowness) in the upper 12.5 m at the I10–La Cienega site. The bridge support columns at I10–La Cienega are located in these low-velocity sediments, and Caltrans engineers have estimated the resonant period of the bridge structures to be between 0.2–0.5 sec (C. Roblee, written commun., 1997) which is in the range of the higher amplification at I10–La Cienega. We therefore speculate that amplified shaking caused by soft ground conditions contributed to the damage and collapse of the highway structures at this site.

## Acknowledgments

We thank Bob Darragh and Tony Shakal of *California Division of Mines and Geology* for permission to make borehole measurements prior to installation of instruments at the La Cienega seismic array, and Cliff Roblee and others at *California Department of Transportation* for engineering data and access to the site. We thank Bob Westerlund for help in the field, and Tom Brocher, Bob Brown, Ken Campbell, Bob Darragh, John Ebel, and an anonymous reviewer for helpful comments on the manuscript, and Bob Simons for his wonderful program CoPlot. John Evans provided a file with the Field and Hough (1997) results. The linear site amplifications were computed using the program NRATTLE, written by Charles Mueller with modifications by Bob Herrmann.

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## Appendix

### Spatial Variability of Peak Accelerations from the 1994 Northridge Earthquake

This Appendix contains a summary of an analysis by Boore (1997) of the spatial variability of peak motion from the 1994 Northridge mainshock. The spatial variability in ground motions reduces to zero as the distance between two sites decreases to zero. On the other hand, for a great enough separation distance the spatial correlation of the ground motions reduces to zero and the additional uncertainty reaches that for an individual observation about the overall change of motion with distance (as given, for example, by fitting the data to a function using regression analysis). The two end-member cases suggests the following equation for the variance of peak ground motions as a function of intersite spacing (because ground motions are well-approximated by a lognormal distribution, the standard deviations in the following discussion are those of the log of the ground motion; uncertainty ranges for the ground motion are given by multiplying and dividing the ground motion by 10 to the standard deviation):

$$\sigma_{\Delta \log Y}^2 = \sigma_{\text{indobs}}^2 \left(1 + \frac{1}{N}\right) F(\Delta)^2, \quad (\text{A1})$$

where  $\sigma_{\Delta \log Y}$  is the standard deviation of differences in the logarithm of the peak motion  $Y$ ,  $\sigma_{\text{indobs}}$  is the standard deviation of an individual observation about a regression, and  $N$  is the number of recordings used in the average of a group of recordings in a small region (the term in  $N$  accounts for the uncertainty in the estimate of the mean motion; for example, if one observation is available and the equation is to be used to compute how much another peak motion might vary as a function of spacing,  $N = 1$ ).  $F(\Delta)$  is a function that accounts for the spatial correlation of the motion, where  $\Delta$  is the average separation between sites;  $F$  takes on values of 0.0 and 1.0 for  $\Delta = 0$  and  $\Delta = \infty$ , respectively.

$F(\Delta)$  was estimated by studying larger peak horizontal accelerations from the 1994 Northridge mainshock, supplemented by studies of spatial variability in small arrays (Abrahamson and Sykora, 1993), the SMART 1 array in Taiwan (Abrahamson, written commun, 1995), and local regions in the 1971 San Fernando earthquake (McCann and Boore, 1983). The analysis for the Northridge data followed these steps:

1. Compute  $\Delta$  for all pairs of stations, keeping only those for which the separation was less than 10 km (over 600 pairs).
2. For each pair, compute the difference of the logarithm of larger peak horizontal acceleration after correcting for differences in distance from the station to the earthquake (the distance attenuation used for this correction was derived in the course



of the analysis as corrections to the average attenuation of Boore *et al.*, 1997, although the results are insensitive to the particular attenuation equation that was used).

3. Divide the range of  $\Delta$  into bins such that 15 station pairs are within each bin. This was done so that a reasonable estimate of the variance of the residuals could be obtained for each bin.
4. Compute the standard deviation of the residuals within each  $\Delta$  bin.
5. Plot the standard deviations against the median distance for each bin, and fit a function to this plot, guided also by the Abrahamson and Boore and McCann studies. The results are shown in Figure 14. This procedure yielded the following equation for  $F(\Delta)$ :

$$F = (1 - \exp - \sqrt{0.6\Delta}). \quad (A2)$$

Regression analyses of data for earthquakes with magnitudes between 6.0 and 6.9 finds that the within-earthquake standard deviation of individual observations about the mean ( $\sigma_{\text{indobs}}$ ) is 0.188 and 0.182 for the larger and random horizontal peak acceleration, respectively (W. Joyner, personal communication, 1996). The results in Figure 14 are shown for the larger peak acceleration.

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## Figure Captions

Figure 1. Location of the borehole at La Cienega Boulevard and Interstate 10 relative to the Northridge earthquake epicenter. The borehole is located at 34.0364N and 118.3780W (NAD83 datum). The Saturn Street School site (USC91) and two other sites (BWH and CCN) from which data are used in this paper are indicated by the triangles.

Figure 2. S-wave recordings for one horizontal direction for the surface-to-borehole logging done at I10–La Cienega. The waveforms have been rotated and filtered with a 30 Hz low-cut Butterworth causal filter. The amplitudes have been individually scaled; the scaling changes when the spacing between recordings changed from 2.5 m to 5.0 m at 100 m. The times in the bottom panel starts at 0.2 sec, but the scaling of time is the same in both panels. The times are relative to the impact at the source, offset 5 m horizontally from the borehole. The heavy solid line is the calculated travel time from the model obtained by fitting first arrivals picked on the waveforms.

Figure 3. Simplified lithology for the core borehole located approximately 4 m south of the hole logged for velocities. The upper 80 meters are nonmarine sediments. The top 5 m have an average S-velocity of 163 m/sec and the average S-velocity to a depth of 12.5 m is 208 m/s. Shear-wave velocity profiles (suspension logging data: dotted; s2b logging: solid line) are shown with two electric logs. The upper portion (23 m, 75 ft) of the borehole had to be surface-cased to prevent cave-in while drilling, precluding electric logs and suspension logging from obtaining data in the top part of the borehole (but suspension logging measurements were made at these shallower depths in a nearby borehole; see Figure 4).

Figure 4. Slowness from suspension log data, compared to layered models fit to that data using two layered models: “s2b” (= **s**urface-to-**b**orehole) uses the layering derived from the analysis of the surface-to-borehole logging of a borehole at I10, and “cmplx” uses more detailed layering guided by the suspension log data. a) I10-La Cienega data models and b) Saturn Street School data and models. Models have the same depth interfaces below 100 m. The suspension log data were obtained from <http://geoinfo.usc.edu/rosrine> and from C. Roblee (written commun., 1999).

Figure 5. Comparison of slowness models determined from surface-to-borehole and suspension log data. The black and gray lines are models for I10 and Saturn Street School, respectively. Two models based on the suspension log data are used, using different layering (see caption of Figure 4 for explanation of “s2b” and “cmplx”). The upper panel gives details within 15 m of the surface, where the various models differ the most. These

differences control the variations in amplification. Although not obvious in the figure, the curves for all but the I10: surface-to-borehole log are the same below about 105 m.

Figure 6. Shear-wave damping  $D_s$ , multiplied by 100 (to give percent damping;  $D_s = 0.5/Q_s$ , where  $Q_s$  is the quality factor), averaged over a depth range of 0 to 220 m, calculated from the decay of amplitude with depth. Horizontal lines represent  $D_s$  derived under the assumption of frequency independent damping. Two suites of waveforms used to derive the damping were obtained, one by driving the surface shear-wave source in one direction (“toward”) and the other by driving the source in the opposite direction (“away”). The near horizontal trend of the values support the constant  $D_s$  assumption over most of the frequency range. The average of the values represented by the two horizontal lines was used in the site amplification calculations.

Figure 7. Site amplification at I10-La Cienega and at Saturn Street School. Graphs a) and b) are for different layered models at the two sites (see caption of Figure 4 for explanation of “s2b” and “cmplx”). Most of the amplification occurs between 1-10 Hz.

Figure 8. Site amplification at I10-La Cienega (SB2) and at Saturn Street School. The top 12.5 m of sediments have been removed to compare the amplification response. Note change in vertical scale compared to Figure 7. The spectra show that the amplification from sediments below 12.5 m at the two sites is very similar.

Figure 9. 5%-damped pseudo relative response spectra for ground motions at the I10 site derived using linear and nonlinear soil response calculations, divided by the similar response spectrum of the recorded motion at the Saturn Street School site. The hatched and gray areas indicate the range of ratios; the areas are generally bounded by models in which the I10 slowness is from the surface-source downhole-receiver (s2b) model (bottom of hatched and gray areas) and from the suspension log data (top of hatched and gray areas). As shown in Figure 5, the suspension logging slownesses are higher than the s2b slownesses near the surface, and that is why the ratio of site response is higher for the I10 model based on the suspension logging data. In all cases the response at the I10 site is systematically higher than that at the Saturn Street School (SAT) site for periods between 0.1 and 1 sec. The resonant period of the bridge structure at I10–La Cienega is estimated to lie between the vertical gray lines (C. Roblee, written commun., 1997).

Figure 10. 5%-damped pseudo relative response spectra for ground motions at the I10 site derived using linear and nonlinear soil response calculations. The hatched and gray areas indicate the range of ratios; the areas are generally bounded by models in which the I10 slowness is from the surface-source downhole-receiver (s2b) model (bottom of hatched

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Figure 11. 5%-damped pseudo relative response spectra for ground motions at Saturn Street School, Baldwin Hills, and Century City – LACC North. The latter two sites have been corrected for geometrical spreading to the distance to Saturn Street School, using the equations of Boore *et al.* (1997). Shown are the geometrical means of the two horizontal components of the spectra computed from the motions recorded at each station. The distance from each station to the I10– La Cienega are given in parenthesis.

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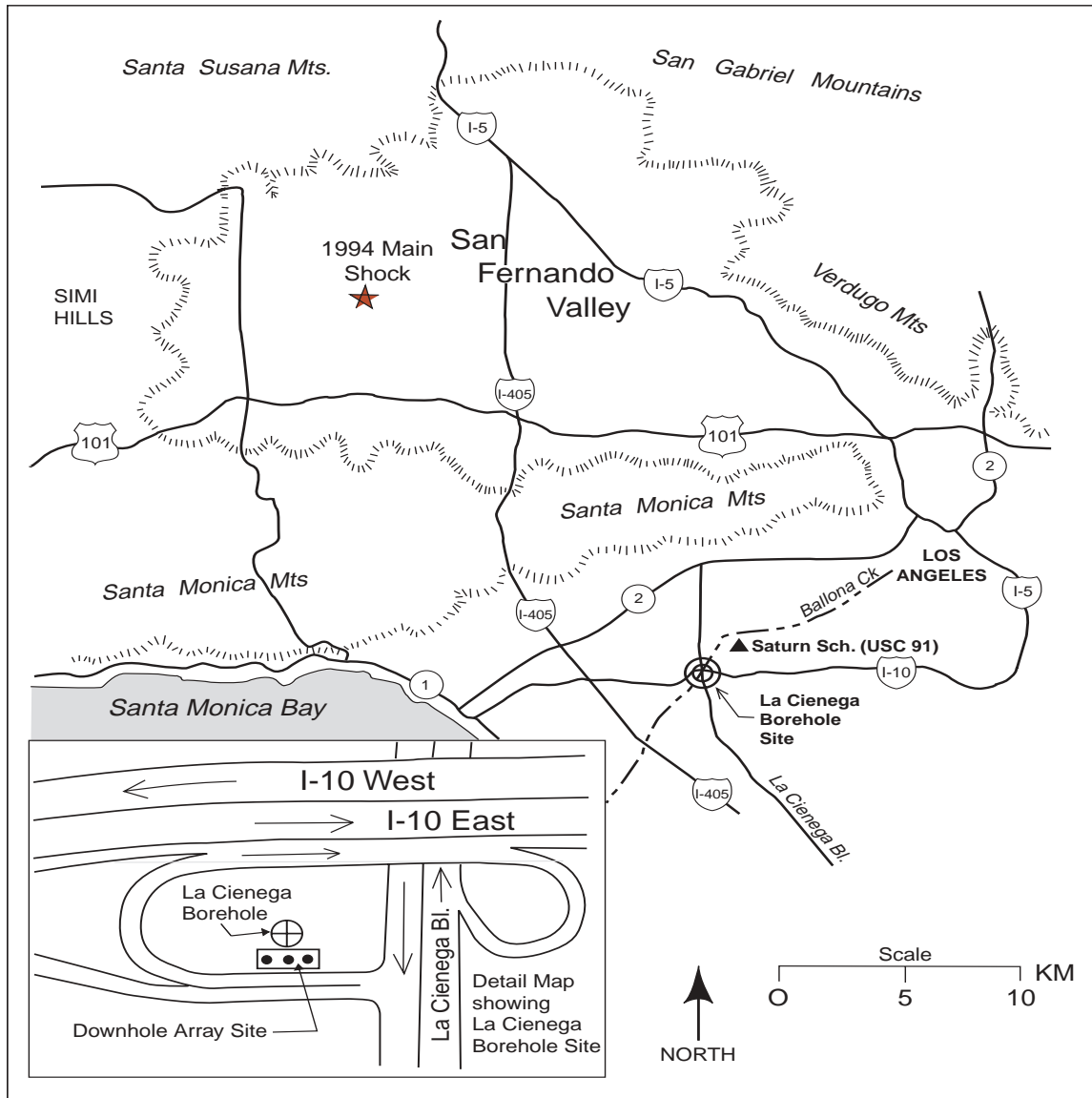


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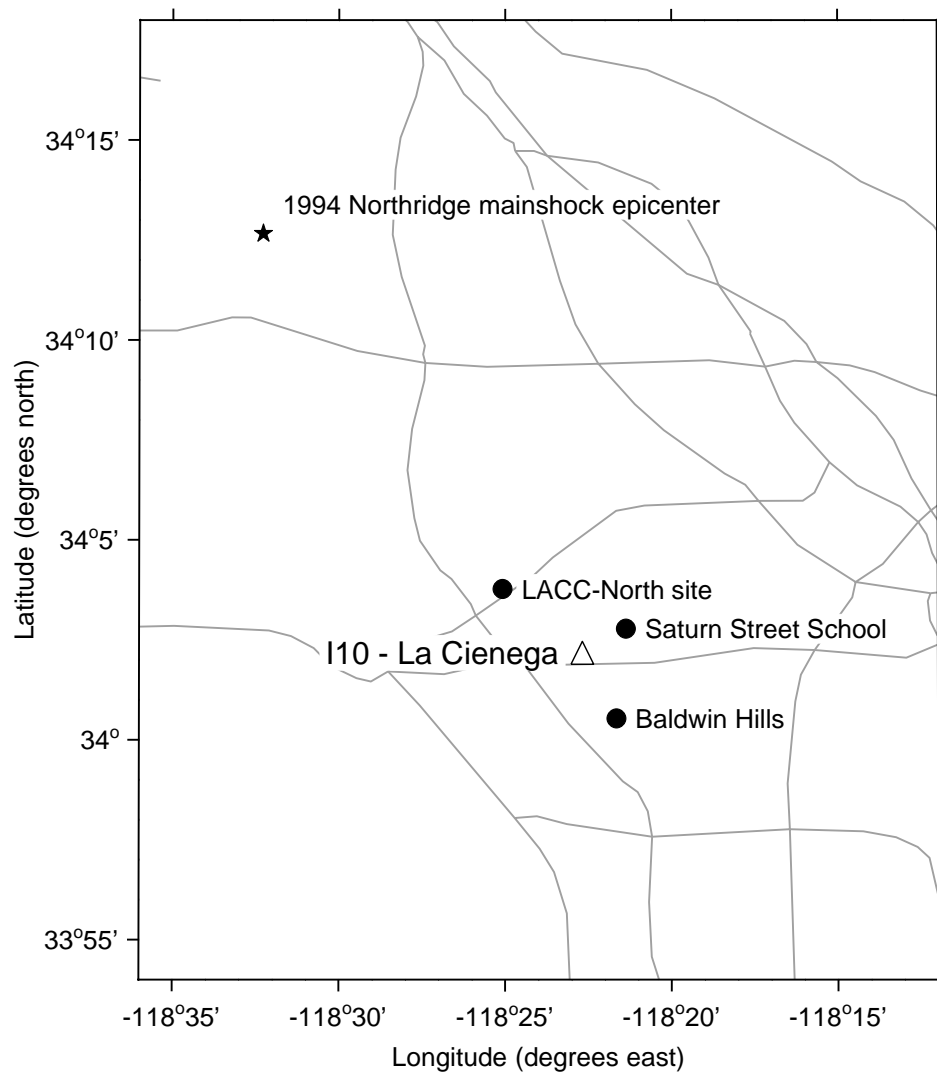


Figure 1. (The station BWH and CCN will be added to the previous figure in the final version of the figure).

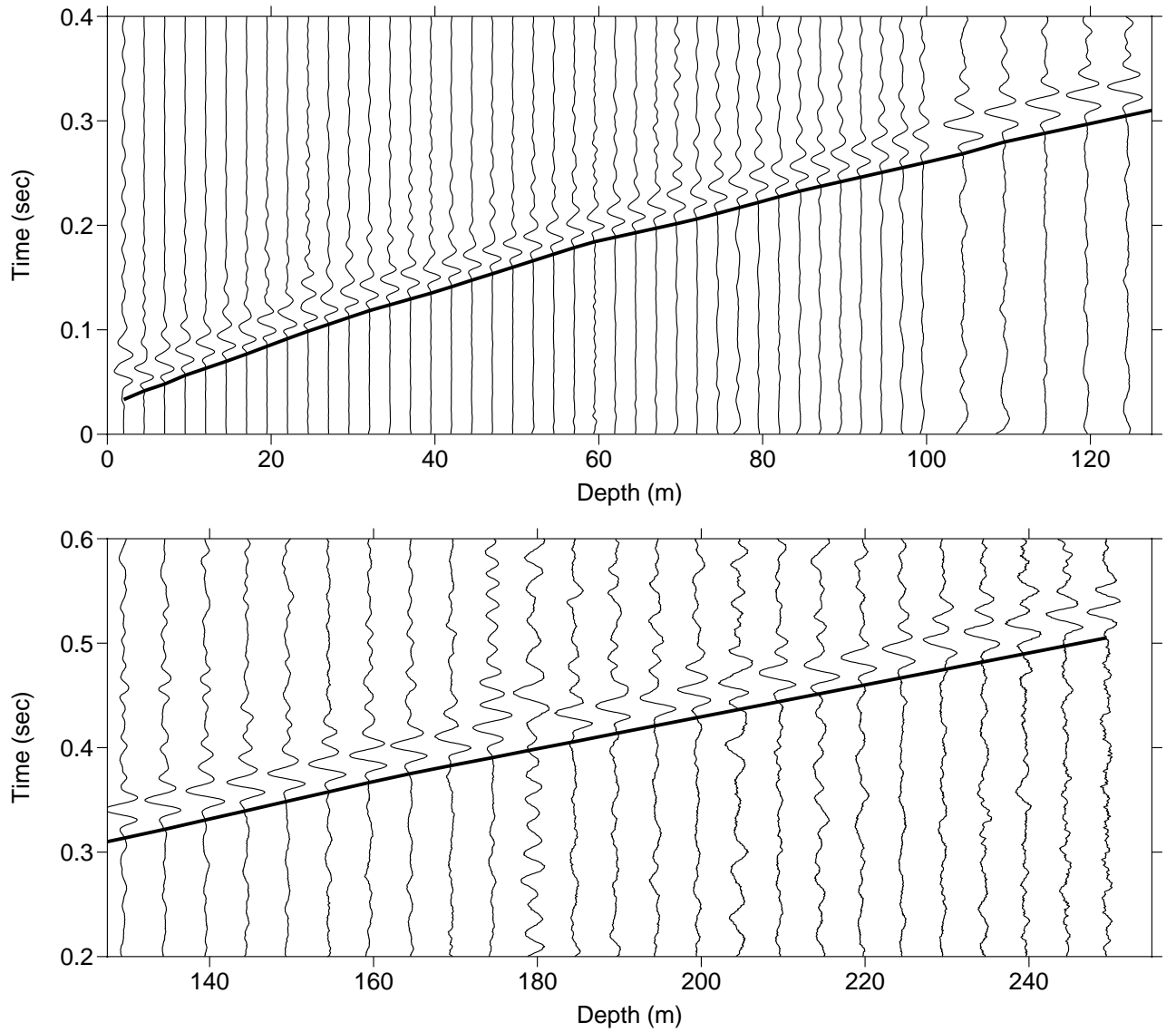


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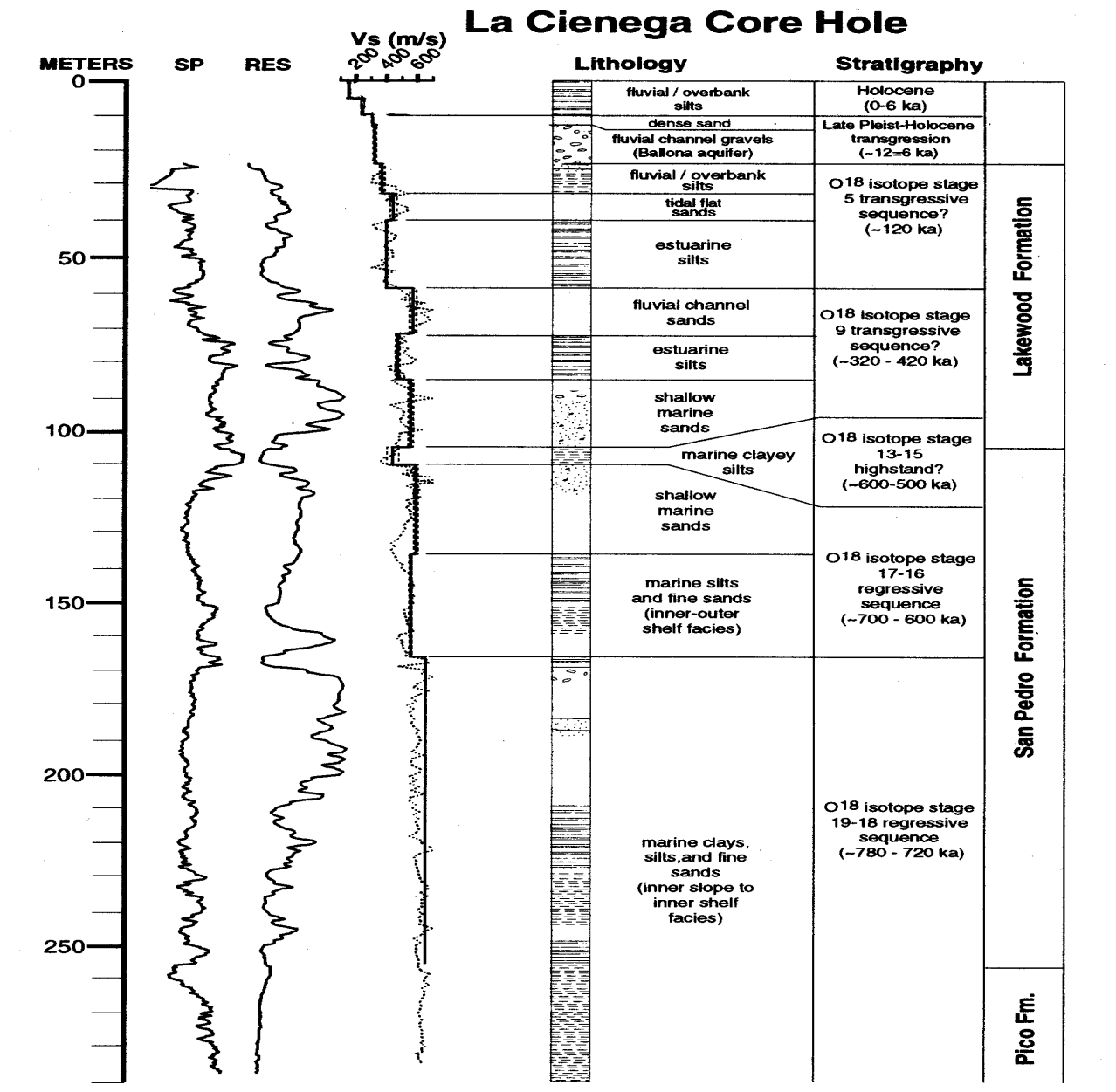


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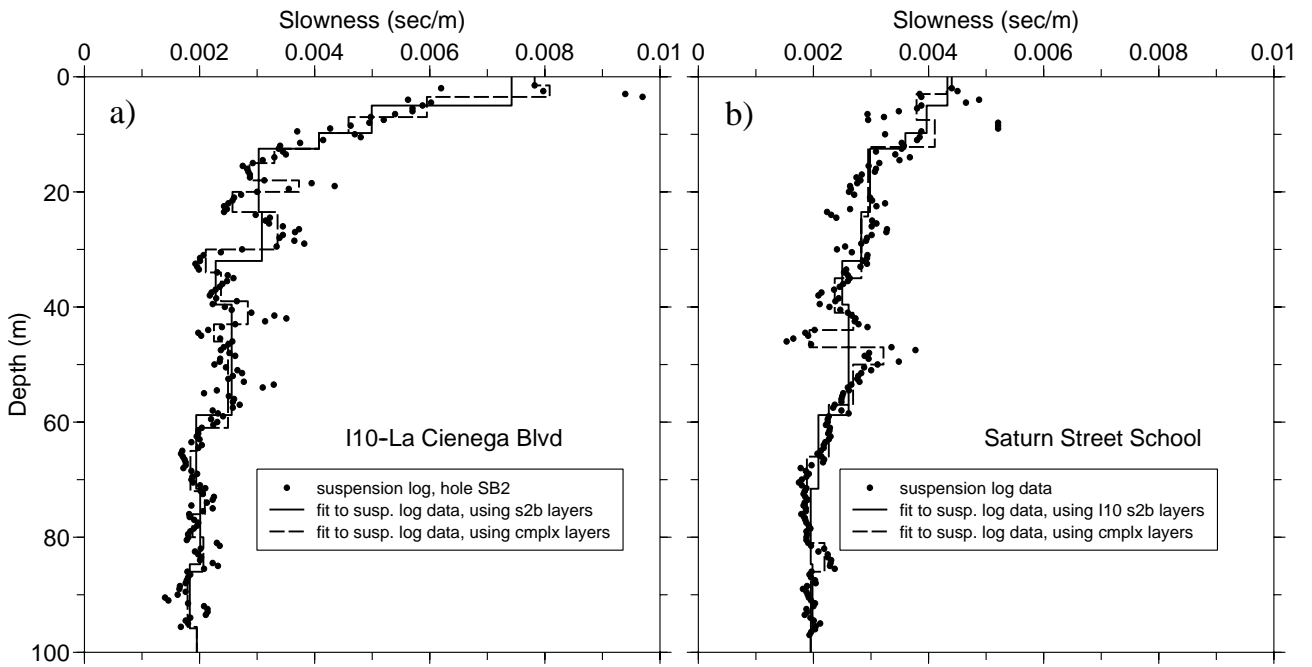


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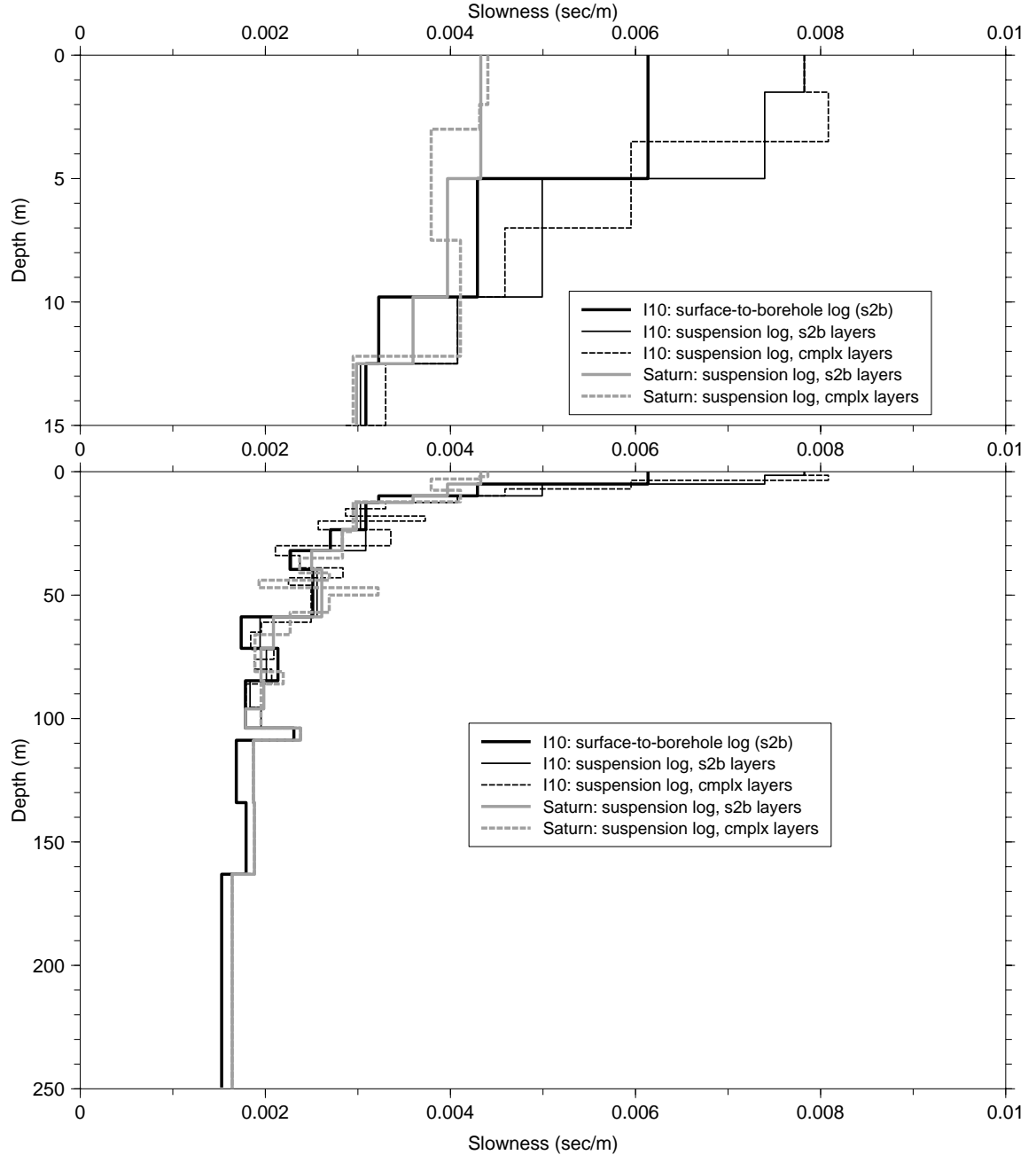


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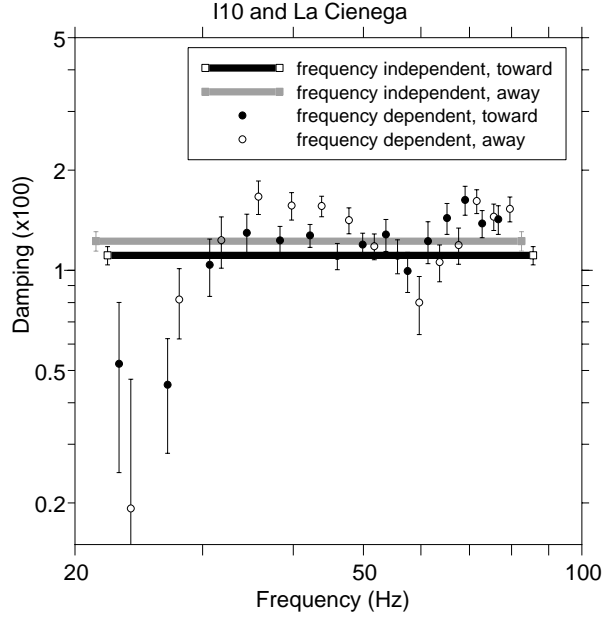


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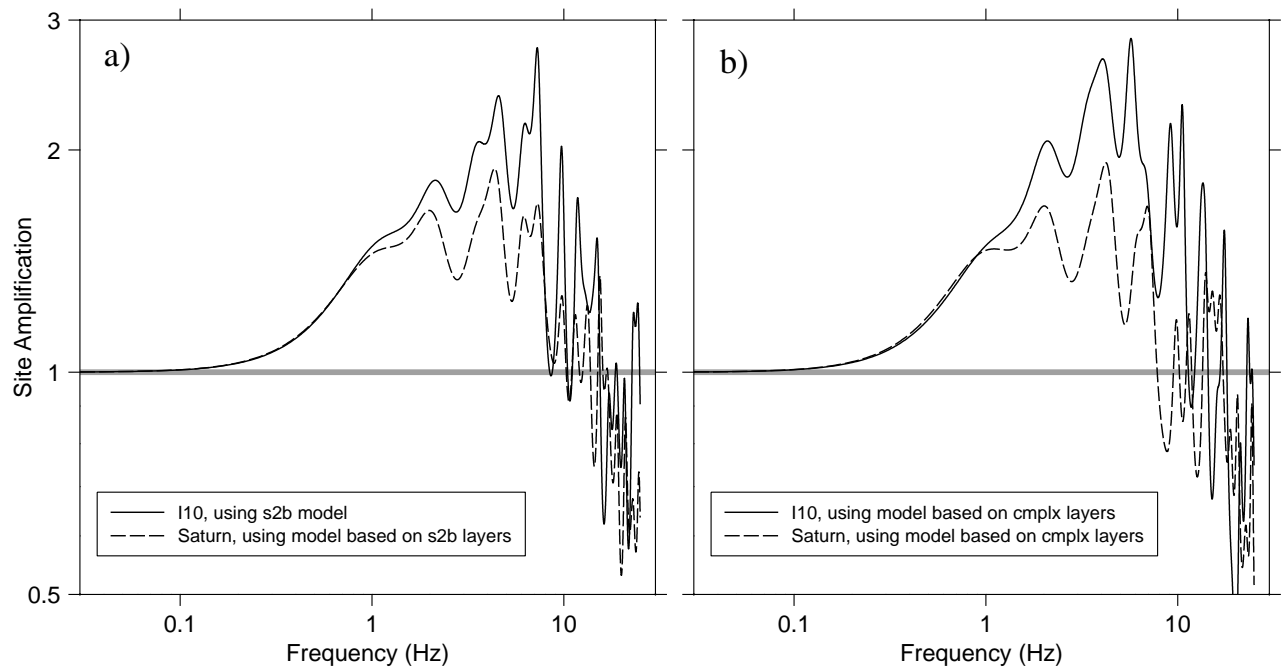


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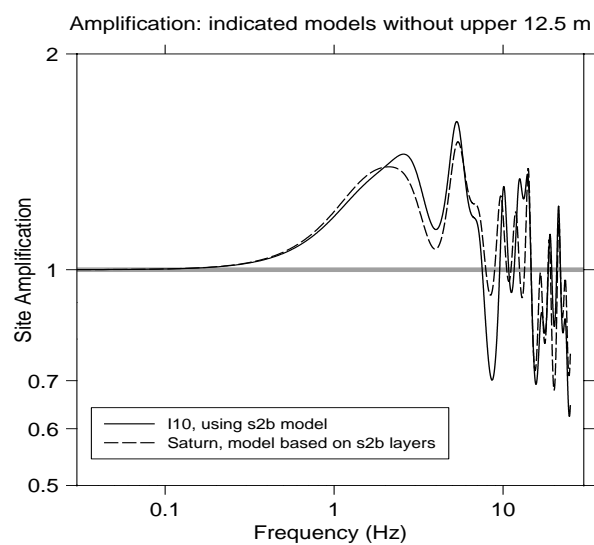


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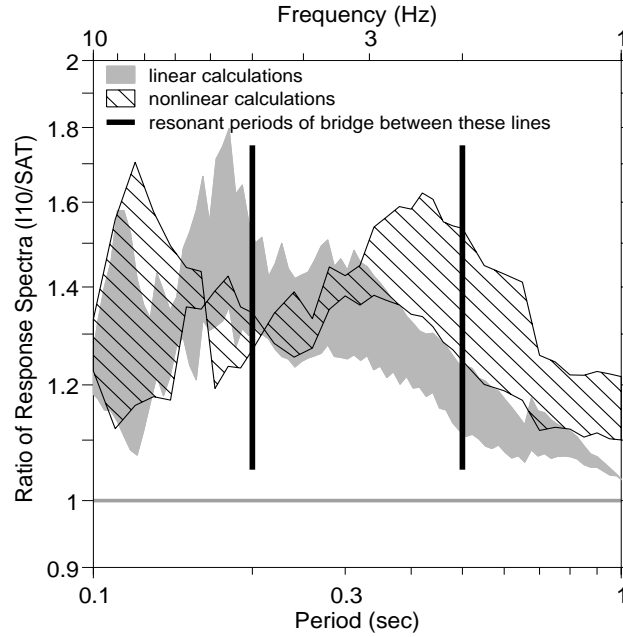


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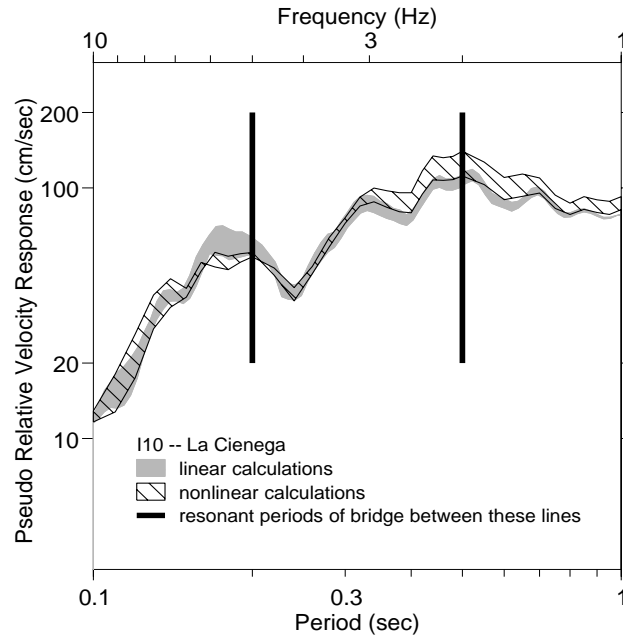


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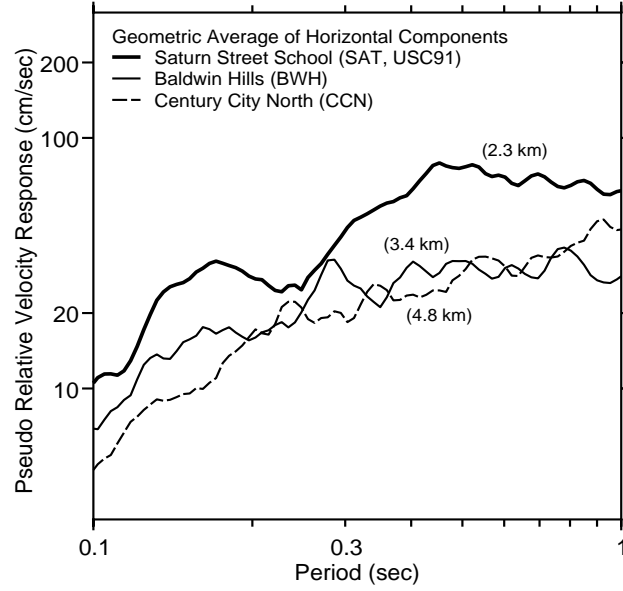


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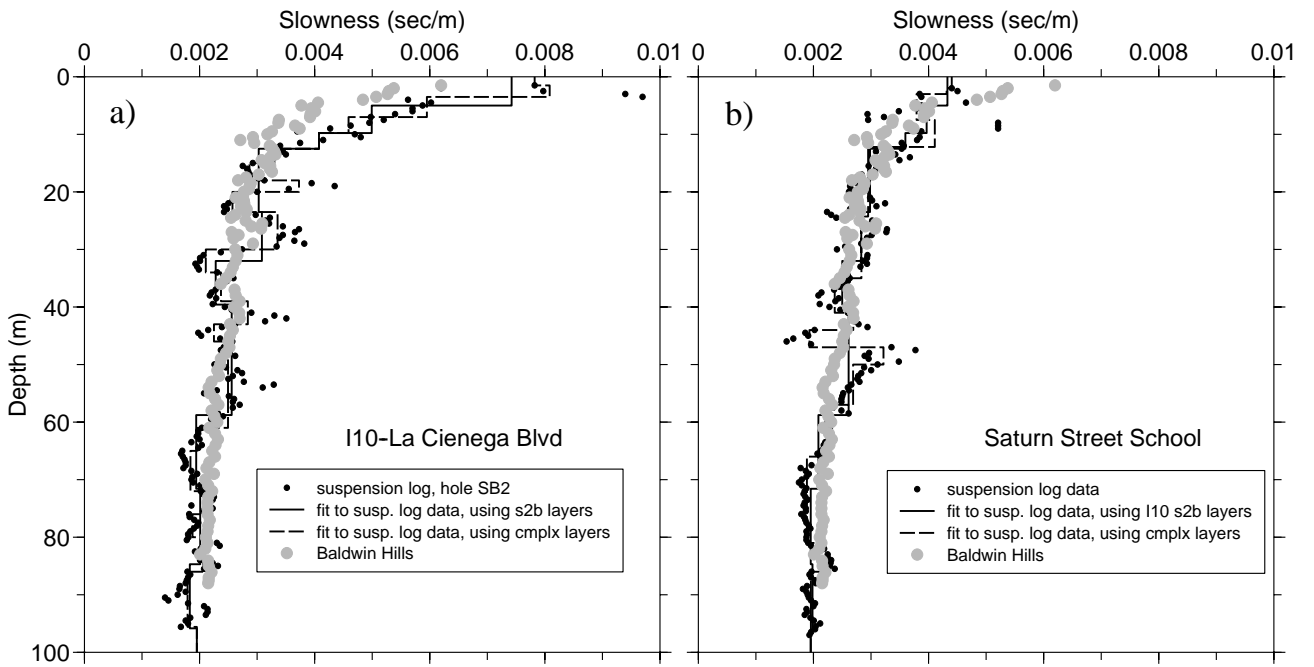


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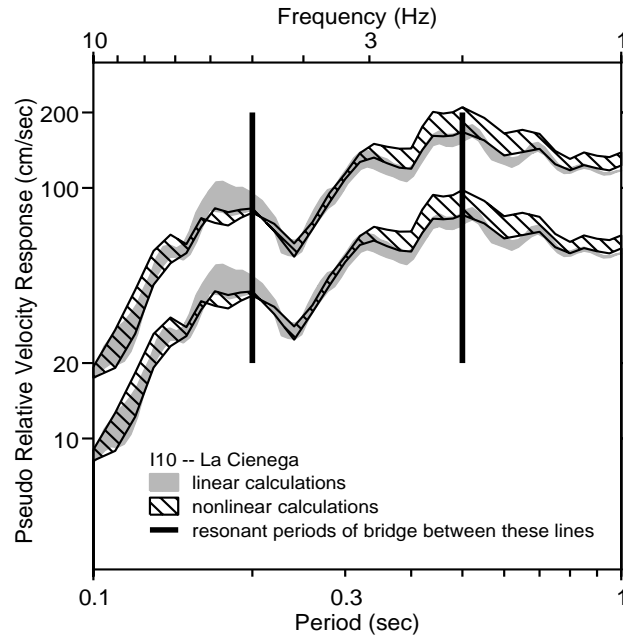


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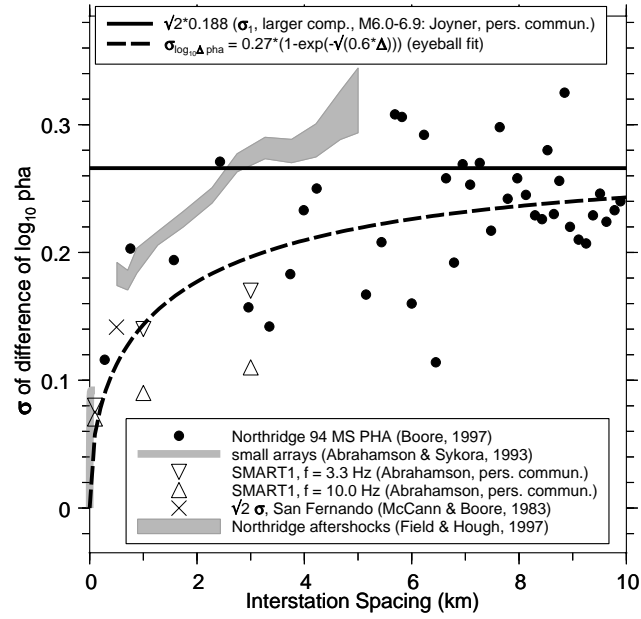


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